

Mathematical Analysis - List 17

1. Find the differential of the function at the given point.

a) $F(m, r) = \frac{Gm}{r^2}$ at $(100, 10)$; b) $Q(K, L) = 1.02K^{0.75}L^{0.25}$ at $(100, 1)$;

c) $f(x, y) = xe^{-2y}$ at $(2, 0)$; d) $h(x, t) = e^{-3t} \sin(x + 5t)$ at $(\pi/2, 0)$.

2. Find the differential of $f(x, y) = \sqrt{x^2 + y^3}$ at the point $(1, 2)$. Use it to estimate $f(1.04, 1.98)$.

3. Find the differential of $f(x, y) = x^y$ at the point $(2, 3)$. Use it to estimate $f(2.04, 2.97)$.

4. A pendulum of period T and length l was used to determine g from the formula

$$g = \frac{4\pi^2 l}{T^2} \quad \text{and} \quad l = s + \frac{k^2}{s}, \quad k < s.$$

If the measurements of k and s are accurate to within 1%, find the maximum percentage error in l . If the measurements of T is accurate to within 0.5%, find the maximum percentage error in the computed value of g .

5. Find the local maxima, local minima of the given function.

a) $f(x, y) = (x + y)(xy + 1)$;

b) $f(x, y) = 1 - \cos x + y^2/2$;

c) $f(x, y) = x^3 + y^3 - 3xy$.

6. What can you conclude about the behavior of f near the point (a, b) if:

a) $f_x = f_y = 0$ at (a, b) and $f_{xx} > 0, f_{yy} > 0, f_{xy} = 0$;

b) $f_x = f_y = 0$ at (a, b) and $f_{xx} > 0, f_{yy} = 0, f_{xy} > 0$;

c) $f_x = f_y = 0$ at (a, b) and $f_{xx} > 0, f_{yy} < 0$.

7. A missile has a remote guidance device which is sensitive to both temperature and humidity. If t is the temperature in $^{\circ}\text{C}$ and h is percent humidity, the range over which the missile can be controlled is given by:

$$R = 27800 - 5t^2 - 6ht - 3h^2 + 400t + 300h.$$

What are the optimal atmospheric conditions for controlling the missile?

8. An irrigation canal has an isosceles trapezoidal cross-section of area 50 m^2 . The average flow rate in the canal is inversely proportional to the wetted perimeter, p , of the canal, that is, to the perimeter of the trapezoid, excluding the top. Thus to maximize the flow we must minimize p . Find the depth d , base width w , and the trapezoid's base angle α that give the maximum flow rate.

9. Find the minimum and maximum values of the function $f(x, y) = 4x^2 - xy + 4y^2$ over the closed disk $x^2 + y^2 \leq 2$.

10. Find the minimum and maximum values of the function $f(x, y) = \sin x + \sin y + \sin(x + y)$ over the closed square $0 \leq x \leq \frac{\pi}{2}, 0 \leq y \leq \frac{\pi}{2}$